Estimation of state-space models for affective dynamics using Markov chain Monte Carlo methods

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Outline

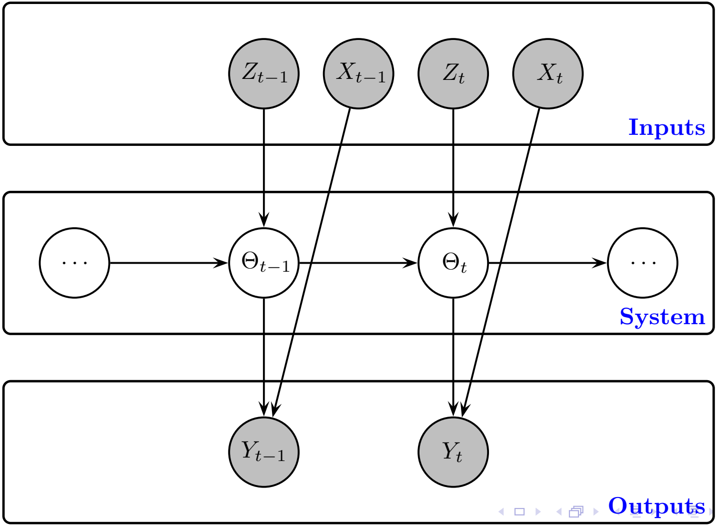
Outline

Introduction

Linear dynamical systems theory

- *Inputs:* observed noise variables / control variables
- *System:* unobserved variables, “states”, that have certain dynamical properties
- *Outputs:* observed variables of interest

Introduction



Introduction

Goal

- *Statistical framework:* Discrete-time state-space models
- *Conceptual framework:* Psychophysiological processes

  $\Rightarrow$ Combine these frameworks

Outline

Oregon adolescent interaction data

Experimental context

- 72 depressed, 69 normal adolescents
- Adolescent, father and mother in laboratory
- Perform interaction tasks
- Physiological measures adolescent
- Interactions videotaped, behavior coding system

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Focus

- Problem solving task, unpleasant (pocket money)
- 18 minutes, measurements each second ($n = 1080$)

Oregon adolescent interaction data

Physiological measures (continuous)

- Heart rate (HR)
- Respiratory sinus arythmia (RSA)
- Skin conductance level (SCL)

Oregon adolescent interaction data

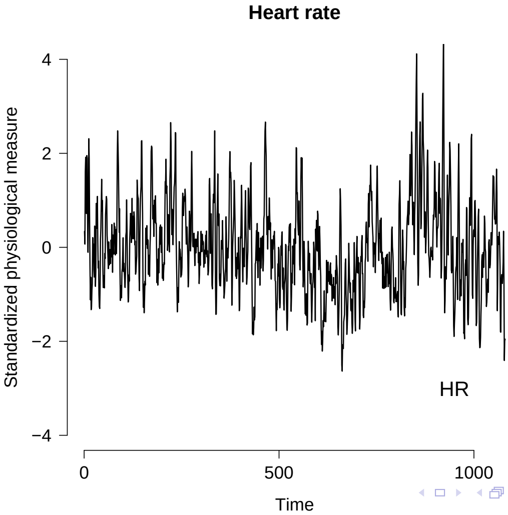
Physiological measures (continuous)

- Heart rate (HR)
- Respiratory sinus arythmia (RSA)
- Skin conductance level (SCL)

Behavioral measures (discrete)

- Anger adolescent (AdAnger)
- Anger father (FaAnger)
- Anger mother (MoAnger)

Oregon adolescent interaction data



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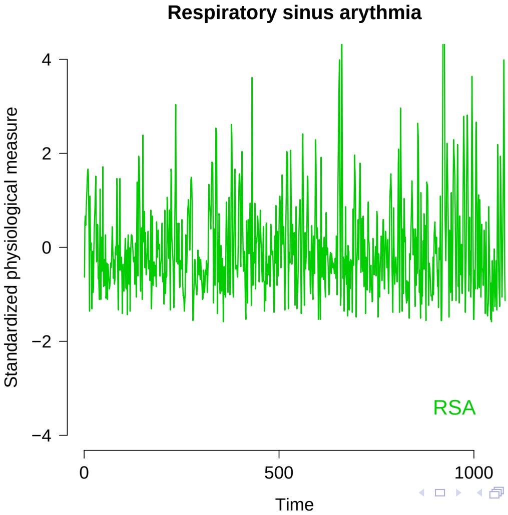

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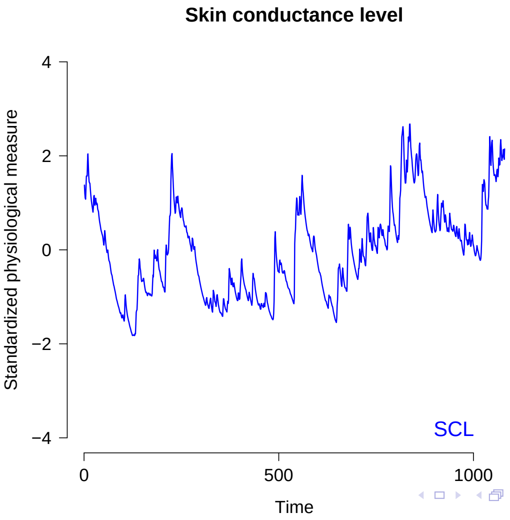

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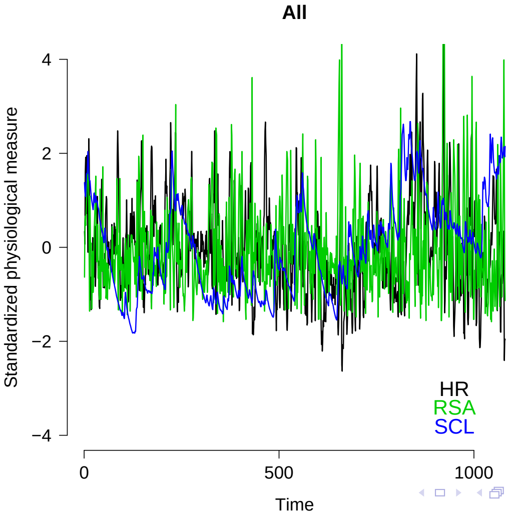

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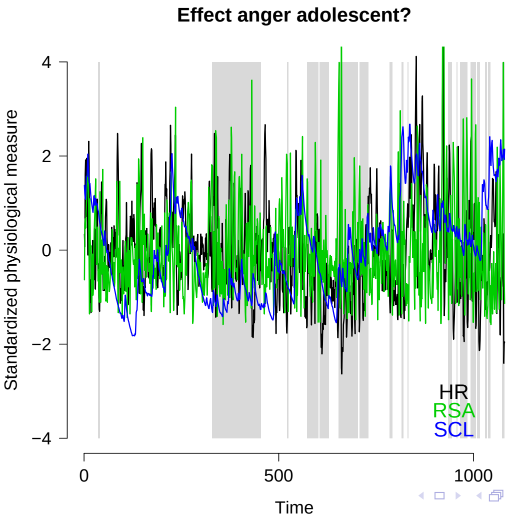

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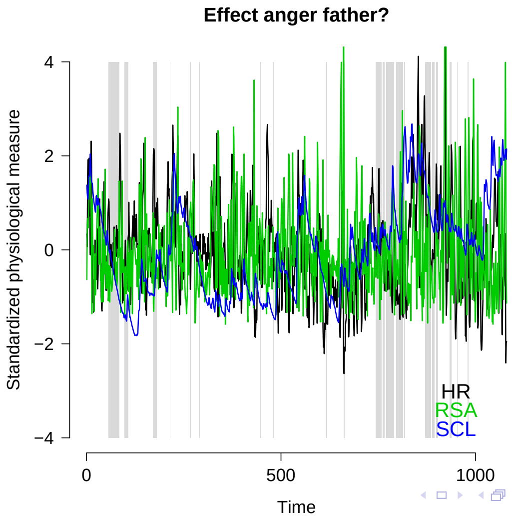

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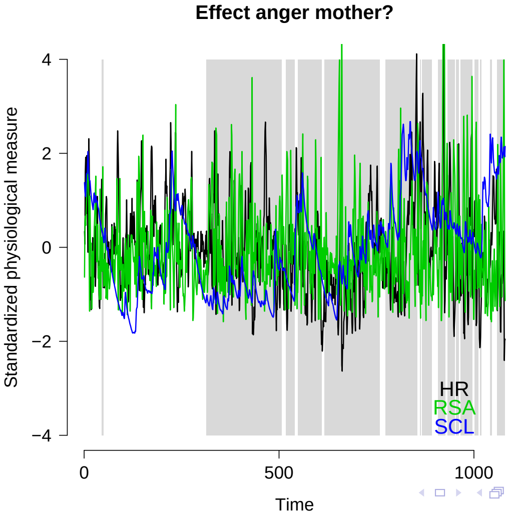

Outline

Multivariate linear Gaussian state-space model

Application fields

- Engineering
- Econometrics
- ...

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Goals

- *State estimation:* True position of a space craft?
- *Parameter estimation:* Dynamical properties emotion system?

Multivariate linear Gaussian state-space model

Transition equation

$$Y_t \sim \text{Gaussian}\left(\Psi\Theta_t + \Gamma X_t, \Sigma_\epsilon\right)$$

$$\Theta_t \sim \text{Gaussian}\left(\Phi\Theta_{t-1} + \Delta Z_t, \Sigma_\eta\right)$$

- Θ_t = states
- Θ_{t-1} = states (previous observation moment)
- Z_t = state covariates

Multivariate linear Gaussian state-space model

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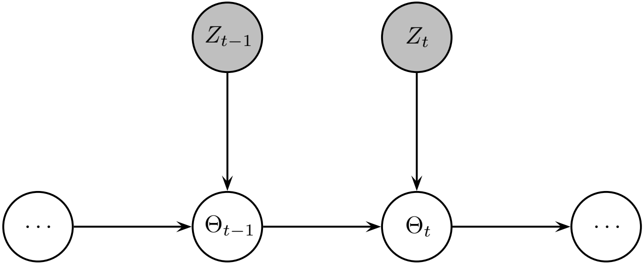
- Θ_t = states
- Θ_{t-1} = states (previous observation moment)
- Z_t = state covariates
- Φ = transition matrix (autocorrelations & cross-lagged relations)
- Δ = state covariate regression coefficient matrix
- Σ_η = innovation variance/covariance matrix

Multivariate linear Gaussian state-space model



Multivariate linear Gaussian state-space model

Observation equation

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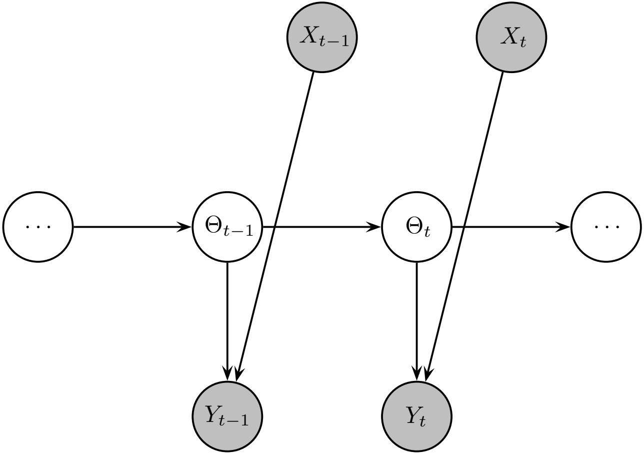
- Y_t = observations
- Θ_t = states
- X_t = observation covariates
- Ψ = design matrix (mapping $Y_t \rightarrow \Theta_t$)
- Γ = observation covariate regression coefficient matrix
- Σ_ϵ = observation error variance/covariance matrix

Multivariate linear Gaussian state-space model



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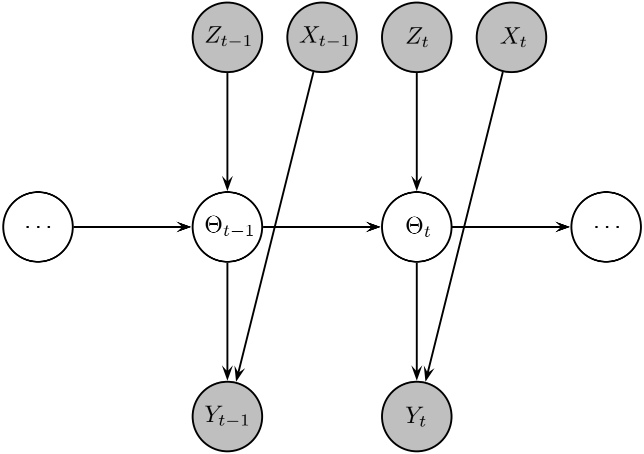

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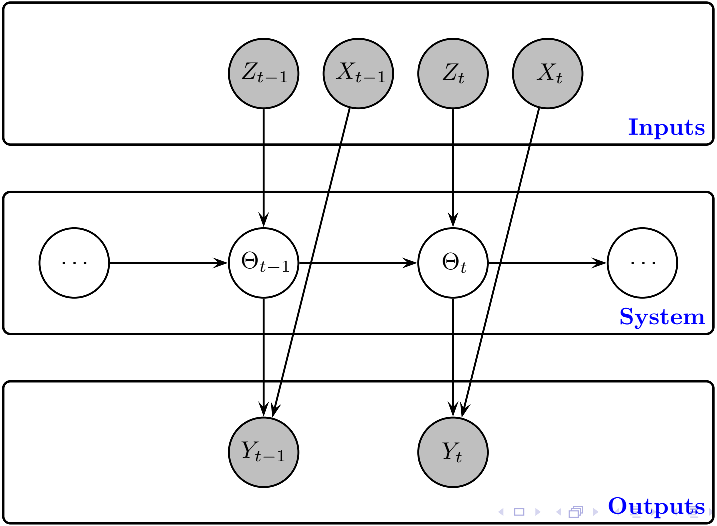

Outline

Regime-switching MLGSS model

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$$Y_t \sim \text{Gaussian}\left(\Psi_t \Theta_t + \Gamma_t X_t, \Sigma_{\epsilon t}\right)$$

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$$\left(\begin{array}{c} \Psi_t, \Gamma_t, \Sigma_{\epsilon t} \\ \Phi_t, \Delta_t, \Sigma_{\eta t} \end{array}\right) = \begin{cases} \left(\begin{array}{c} \Psi_1, \Gamma_1, \Sigma_{\epsilon 1} \\ \Phi_1, \Delta_1, \Sigma_{\eta 1} \end{array}\right) & \text{if } R_t = 1 \\ \qquad \ldots & \\ \left(\begin{array}{c} \Psi_r, \Gamma_r, \Sigma_{\epsilon r} \\ \Phi_r, \Delta_r, \Sigma_{\eta r} \end{array}\right) & \text{if } R_t = r \end{cases}$$

- Dynamical properties $<$ current regime R_t
- Regime variable R_t observed or unobserved

Outline

Gibbs sampler

Gibbs sampler?

- Bayesian parameter estimation
- Iterative simulation algorithm
- Sampling distribution for each single parameter

Gibbs sampler

Gibbs sampler?

- Bayesian parameter estimation
- Iterative simulation algorithm
- Sampling distribution for each single parameter
- Convergence to joint posterior distribution
- Values simulate true joint posterior distribution
- Blocking efficient for some models

Gibbs sampler

Regime-switching MLGSS model

$$Y_t \sim \text{Gaussian}\left(\Psi_t \Theta_t + \Gamma_t X_t, \Sigma_\epsilon\right)$$

$$\Theta_t \sim \text{Gaussian}\left(\Phi_t \Theta_{t-1} + \Delta_t Z_t, \Sigma_\eta\right)$$

$$\begin{pmatrix} \Psi_t, \Gamma_t \\ \Phi_t, \Delta_t \end{pmatrix} = \begin{cases} \begin{pmatrix} \Psi_1, \Gamma_1 \\ \Phi_1, \Delta_1 \end{pmatrix} & \text{if } R_t = 1 \\ \ldots & \\ \begin{pmatrix} \Psi_r, \Gamma_r \\ \Phi_r, \Delta_r \end{pmatrix} & \text{if } R_t = r \end{cases}$$

Limitation: No regime-switching for Σ_ϵ & Σ_η

Gibbs sampler

Blocked Gibbs sampler

1. $\mathrm{p}(\Theta \mid \Psi_{1,..,r}, \Gamma_{1,..,r}, \Sigma_\epsilon, \Phi_{1,..,r}, \Delta_{1,..,r}, \Sigma_\eta, Y, X, Z)$
2. $\mathrm{p}(\Psi_{1,..,r}, \Gamma_{1,..,r}, \Sigma_\epsilon \mid \Phi_{1,..,r}, \Delta_{1,..,r}, \Sigma_\eta, \Theta, Y, X, Z)$
3. $\mathrm{p}(\Phi_{1,..,r}, \Delta_{1,..,r}, \Sigma_\eta \mid \Psi_{1,..,r}, \Gamma_{1,..,r}, \Sigma_\epsilon, \Theta, Y, X, Z)$

Outline

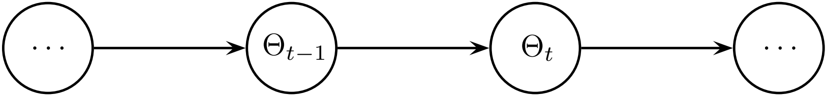
Sampling states

Blocked Gibbs sampler

1. $\mathrm{p}(\Theta \mid \Psi_{1,..,r}, \Gamma_{1,..,r}, \Sigma_{\epsilon}, \Phi_{1,..,r}, \Delta_{1,..,r}, \Sigma_{\eta}, Y, X, Z)$
   $\longrightarrow$ Forward-filtering backward sampling
2. $\mathrm{p}(\Psi_{1,..,r}, \Gamma_{1,..,r}, \Sigma_{\epsilon} \mid \Phi_{1,..,r}, \Delta_{1,..,r}, \Sigma_{\eta}, \Theta, Y, X, Z)$
3. $\mathrm{p}(\Phi_{1,..,r}, \Delta_{1,..,r}, \Sigma_{\eta} \mid \Psi_{1,..,r}, \Gamma_{1,..,r}, \Sigma_{\epsilon}, \Theta, Y, X, Z)$

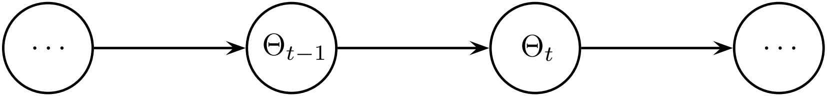
Sampling states

$\Rightarrow$ **Kalman filter** $\Rightarrow$



Sampling states

$\Rrightarrow$ **Kalman filter** $\Rrightarrow$



$\Lleftarrow$ **Backward-sampling** $\Lleftarrow$

Outline

Sampling parameters

Blocked Gibbs sampler

1. $p(\Theta \mid \Psi_{1,..,r}, \Gamma_{1,..,r}, \Sigma_\epsilon, \Phi_{1,..,r}, \Delta_{1,..,r}, \Sigma_\eta, Y, X, Z)$
2. $p(\Psi_{1,..,r}, \Gamma_{1,..,r}, \Sigma_\epsilon \mid \Phi_{1,..,r}, \Delta_{1,..,r}, \Sigma_\eta, \Theta, Y, X, Z)$
   $\longrightarrow$ Exact posterior simulation (decomposition)
3. $p(\Phi_{1,..,r}, \Delta_{1,..,r}, \Sigma_\eta \mid \Psi_{1,..,r}, \Gamma_{1,..,r}, \Sigma_\epsilon, \Theta, Y, X, Z)$

Sampling parameters

Prior distributions

$$\Psi_{1,..,r} \propto 1$$

$$\Gamma_{1,..,r} \propto 1$$

$$\Sigma_\epsilon \sim \mathrm{W}^{-1}(\nu_0, S_0)$$

Sampling parameters

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Decomposition

$$\mathrm{p}(\Psi_{1,..,r}, \Gamma_{1,..,r}, \Sigma_\epsilon \mid \Phi_{1,..,r}, \Delta_{1,..,r}, \Sigma_\eta, \Theta, Y, X, Z)$$

Sampling parameters

Prior distributions

$$\Psi_{1,..,r} \propto 1$$
$$\Gamma_{1,..,r} \propto 1$$
$$\Sigma_\epsilon \sim \mathrm{W}^{-1}(\nu_0, S_0)$$

Decomposition

$$\begin{aligned} & \mathrm{p}(\Psi_{1,..,r}, \Gamma_{1,..,r}, \Sigma_\epsilon \mid \Phi_{1,..,r}, \Delta_{1,..,r}, \Sigma_\eta, \Theta, Y, X, Z) \\ = {} & \mathrm{p}(\Psi_{1,..,r}, \Gamma_{1,..,r}, \Sigma_\epsilon \mid \Theta, Y, X) \end{aligned}$$

Sampling parameters

Prior distributions

$$\Psi_{1,\ldots,r} \propto 1$$
$$\Gamma_{1,\ldots,r} \propto 1$$
$$\Sigma_\epsilon \sim \mathrm{W}^{-1}(\nu_0, S_0)$$

Decomposition

$$\begin{aligned}
&\mathrm{p}(\Psi_{1,\ldots,r}, \Gamma_{1,\ldots,r}, \Sigma_\epsilon \mid \Phi_{1,\ldots,r}, \Delta_{1,\ldots,r}, \Sigma_\eta, \Theta, Y, X, Z) \\
&= \mathrm{p}(\Psi_{1,\ldots,r}, \Gamma_{1,\ldots,r}, \Sigma_\epsilon \mid \Theta, Y, X) \\
&= \mathrm{p}(\Sigma_\epsilon \mid \Theta, Y, X) \rightarrow \text{sample from } \mathrm{W}^{-1} \\
&\times \mathrm{p}(\Psi_{1,\ldots,r} \mid \Theta, \Sigma_\epsilon, Y, X) \rightarrow \text{sample from Gaussian} \\
&\times \mathrm{p}(\Gamma_{1,\ldots,r} \mid \Theta, \Psi_{1,\ldots,r}, \Sigma_\epsilon Y, X) \rightarrow \text{sample from Gaussian}
\end{aligned}$$

Sampling parameters

Blocked Gibbs sampler

1. $p(\Theta \mid \Psi_{1,\ldots,r}, \Gamma_{1,\ldots,r}, \Sigma_\epsilon, \Phi_{1,\ldots,r}, \Delta_{1,\ldots,r}, \Sigma_\eta, Y, X, Z)$
2. $p(\Psi_{1,\ldots,r}, \Gamma_{1,\ldots,r}, \Sigma_\epsilon \mid \Phi_{1,\ldots,r}, \Delta_{1,\ldots,r}, \Sigma_\eta, \Theta, Y, X, Z)$
3. $p(\Phi_{1,\ldots,r}, \Delta_{1,\ldots,r}, \Sigma_\eta \mid \Psi_{1,\ldots,r}, \Gamma_{1,\ldots,r}, \Sigma_\epsilon, \Theta, Y, X, Z)$
   $\rightarrow$ Exact posterior simulation (decomposition)

Outline

Application

Data

- Oregon adolescent interaction study
- 13 depressed, 6 normal participants ($> 10\%$ anger)

Application

Data

- Oregon adolescent interaction study
- 13 depressed, 6 normal participants ($> 10\%$ anger)
- $Y_t = [\text{HR RSA SCL}]_t$

  $Z_t = [\text{FaAnger MoAnger}]_t$

  $R_t = [\text{AdAnger}]_t$

Application

Regime-switching MLGSS Model

- $Y_t \sim \text{Gaussian}\left(\Psi\Theta_t, \Sigma_\epsilon\right)$
  $\Theta_t \sim \text{Gaussian}\left(\Phi_t\Theta_{t-1} + \Delta Z_t, \Sigma_\eta\right)$
  $$\Phi_t = \begin{cases} \Phi_1 \text{ if } R_t = 1 \text{ (no anger)} \\ \Phi_2 \text{ if } R_t = 2 \text{ (anger)} \end{cases}$$
- *Fixed:* $\Psi = I_3$
- *Free:* $\Phi_1, \Phi_2, \Delta, \Sigma_\epsilon, \Sigma_\eta$

Application

$$\hat{\Delta}^{n} = \begin{array}{cc} & \begin{array}{cc} \text{FaAnger} & \text{MoAnger} \end{array} \\ \begin{array}{c} \text{HR} \\ \text{RSA} \\ \text{SCL} \end{array} & \begin{bmatrix} 0 & 0 \\ -.01 & 0 \\ -.01 & 0 \end{bmatrix} \end{array} \qquad \hat{\Delta}^{d} = \begin{array}{cc} & \begin{array}{cc} \text{FaAnger} & \text{MoAnger} \end{array} \\ \begin{array}{c} \text{HR} \\ \text{RSA} \\ \text{SCL} \end{array} & \begin{bmatrix} -.02 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \end{array}$$

$$\hat{\Phi}_{1}^{n} = \begin{array}{cc} \begin{array}{c} \text{HR} \\ \text{RSA} \\ \text{SCL} \end{array} & \begin{bmatrix} .75 & .03 & .03 \\ -.03 & .75 & 0 \\ .01 & 0 & .98 \end{bmatrix} \end{array} \qquad \hat{\Phi}_{1}^{d} = \begin{array}{cc} \begin{array}{c} \text{HR} \\ \text{RSA} \\ \text{SCL} \end{array} & \begin{bmatrix} .81 & 0 & .02 \\ -.05 & .71 & 0 \\ .01 & 0 & .98 \end{bmatrix} \end{array}$$

$$\hat{\Phi}_{2}^{n} = \begin{array}{cc} \begin{array}{c} \text{HR} \\ \text{RSA} \\ \text{SCL} \end{array} & \begin{bmatrix} .74 & .03 & .02 \\ -.06 & .69 & -.01 \\ .04 & 0 & .98 \end{bmatrix} \end{array} \qquad \hat{\Phi}_{2}^{d} = \begin{array}{cc} \begin{array}{c} \text{HR} \\ \text{RSA} \\ \text{SCL} \end{array} & \begin{bmatrix} .78 & .02 & .02 \\ -.02 & .67 & 0 \\ .02 & .03 & .98 \end{bmatrix} \end{array}$$

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Application

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Outline

Conclusion

Summary

- Combination straightforward techniques
  $\longrightarrow$ Bayesian estimation algorithm for MLGSS model
- MLGSS model might be useful for investigation psycho(physio)logical processes
  - No differences found between groups / regimes (preliminary)
  - Model extensions might be necessary

Conclusion

Future work

- Proper prior distributions
- Regime-switching Σ_ϵ & Σ_η
- Hierarchical extension
- Non-Gaussian error distributions
- Model selection